Born–Infeld action for gravitational and electroweak fields

Dmitriy Palatnik[*]

Abstract

This note suggests a generalization of the Born–Infeld action (1932) on the case of electroweak and gravitational fields in 4-dimensional spacetime. Basic notions one deals with are Dirac matrices, γ_a, and dimensionless covariant derivatives, $\pi_a = -i\ell\nabla_a$. The action, constructed from γ_a and π_a, contains a parameter, ℓ, (which is of order of magnitude of Planck's length), and possesses additional symmetry with respect to transformations of the Lorentz group imposed on pairs (γ_a, π_a). This symmetry leads to the most natural coexistence between standard electroweak and gravitational action terms in the action's expansion in ℓ, and provides basis for unification of the interactions. It's shown that the parameter of the Lorentz group is associated with a constant value of the electroweak potential at spatial infinity.

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[*]mailto: palatnik@ripco.com

I. INTRODUCTION

> The present educated view of the standard model, and of general relativity, is ... that these are the leading terms in effective field theories.
>
> S. Weinberg [8]

Development in string models has led to rediscovery of the Born–Infeld (BI) theory [1–3], initially formulated for electromagnetic fields. It was shown [4–7], that in the theories of open strings the low-energy effective actions coincide with the BI one. As a result, research in both electromagnetic and gravitational BI gained new momenta.

Nonlinear modifications of the standard gravitational and electromagnetic actions have been studied with different corrections. For example, couplings of the BI electromagnetic action to the Einstein–Hilbert (EH) one have been considered by Gibbons and Herdeiro [9], by Wirschins et al. [10], by Ayón-Beato and García [11–13], by Chang et al. [14]. Nonlinear in curvature tensor gravitational actions of BI type have been studied by Feigenbaum et al. [15], by Wohlfarth [16], by Nieto [17], and also in [18].

The motivation for original BI theory was the possibility to obtain finite energy solutions to the BI field equations, which should describe, according to the authors, structure of an electron. Having that in mind, one attempts to construct from scratch (i.e., with no reference to any string model) a BI action for gravitational and electroweak fields ('BI-gew'), which should, hopefully, lead to solutions, corresponding to field condensations with finite energy.

An essential feature of BI-gew, suggested below, is nonlinearity of the action in curvature tensor, ρ_{ab}, which substitutes, roughly speaking, dimensionless electromagnetic tensor, F_{ab}, in BI invariant, $\int d\Omega \sqrt{-\det(\eta_{ab} + F_{ab})}$. Due to this nonlinearity one obtains both linear and quadratic in ρ_{ab} terms in the action's expansion, corresponding, respectively, to the EH term for gravity and to the standard one for electroweak fields.

As an asset of BI-gew (as well as of original BI), one may consider the fact that action invariants contain only one dimensional parameter, ℓ, which fixes coefficients of the action's expansion in powers of the curvature tensor.

One may specify yet another reason for considering the BI-gew action; namely, its additional (to general covariance, gauge invariance, and invariance under matrix transformations) symmetry with respect to transformations of Lorents group imposed on pairs (γ_a, π_a),* where γ_a are the Dirac matrices and π_a are dimensionless covariant derivative operators. The symmetry is introduced 'by hands' and makes BI-gew look more appealing aesthetically. Besides aesthetical appeal, one may offer additional argument in favor of selecting a more symmetric action. The higher is the symmetry of the action, the more restricted is the class of invariants. Since the goal of modern research is to find the unified field theory with unique action invariant(s), one should welcome any upgrade of existing theory, leading to higher symmetry of the action, than that of its predecessor.

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* This symmetry is analogous to the symmetry of the (1-dim) action for a relativistic particle, $-\int \sqrt{dt^2 - dx^2}$, invariant with respect to Lorents transformations imposed on pair (dt, dx). One may observe, that the relativistic particle action was the inspiration for original BI one.

One is able to conceal all interactions in connections, associated with π_a. As a result all interactions enter the action on equal footing. One believes, that if unification of the interactions is feasible in four dimensions at all, then given approach may provide basis for such unification.

To begin with, one specifies basic elements of the action, compatible with quantum field theory in 4-dimensional spacetime; namely, (α) Dirac $N \times N$ matrices, γ_a, submitted to relations

$$\gamma_{(a}\,\gamma_{b)} = g_{ab}\hat{1}_N \,, \tag{1.1}$$

where g_{ab} is the metric tensor, and $\hat{1}_N$ is unit $N \times N$ matrix; (β) dimensionless operators

$$\pi_a \;=\; -i\ell\nabla_a \,, \tag{1.2}$$

where ∇_a is a covariant derivative and ℓ is a parameter. Action of (1.2) on spinors, scalars, and Dirac matrices is specified in the following text. For spinors and scalars, for example, one obtains,

$$\pi_a\Psi = -i\ell\left(\partial_a\Psi - \Gamma_a\Psi\right) \;; \tag{1.3}$$
$$\pi_a\Phi = -i\ell\left(\partial_a\Phi - \dot{\Gamma}_a\Phi\right) \;; \tag{1.4}$$

here Ψ (Φ) represents spinor (scalar), and Γ_a ($\dot{\Gamma}_a$) are connection matrices. Introducing $\Psi_a = \gamma_a\Psi$, one obtains,

$$\pi_b\Psi_a = -i\ell\left(\partial_b\Psi_a - \Gamma_{ab}^c\Psi_c - \Gamma_b\Psi_a\right) \,. \tag{1.5}$$

Here $\Gamma_{ab}^c = \Gamma_{(ab)}^c$ is connection symbol.[†] From (1.3) and (1.5) one may easily obtain $[\pi_b, \gamma_a]$. By a postulate, the action is form-invariant with respect to substitutions, $\gamma_a \mapsto \gamma_a'$, $\pi_a \mapsto \pi_a'$, where

$$\gamma_a' = \cosh\theta\,\gamma_a + \sinh\theta\,\pi_a \,, \tag{1.6}$$
$$\pi_a' = \sinh\theta\,\gamma_a + \cosh\theta\,\pi_a \,, \tag{1.7}$$

and θ doesn't depend on coordinates. In section VI the physical meaning of parameter θ is clarified. It is shown that θ is associated with the value of electroweak potential at spatial infinity.

II. OBJECTS OF THE THEORY

1. One may introduce dimensionless curvature tensor,

$$\rho_{ab} = 2\pi_{[a}\pi_{b]} = 2\ell^2\left(\partial_{[a}\Gamma_{b]} - \Gamma_{[a}\Gamma_{b]}\right) \,, \tag{2.1}$$

[†] The connection symbol, Γ_{ab}^c, coincides with the Christoffel one only in leading approximation in ℓ. One's intent is to use the Palatini method in order to find Γ's and γ's.

where Γ_a are connections, given in spinorial or scalar representation. Operator ρ_{ab} is matrix $N \times N$.

2. Define a tensorial operator,

$$\phi_{ab} = \gamma_{[a}\gamma_{b]} - \pi_{[a}\pi_{b]} \,. \tag{2.2}$$

3. Define a scalar density,

$$\phi = \frac{1}{5!\,N}\,e^{abcd}\,e^{efgh}\,Tr\left\{\phi_{ae}\,\phi_{bf}\,\phi_{cg}\,\phi_{dh}\right\}\,, \tag{2.3}$$

where $e^{abcd} = e^{[abcd]}$ is the absolute antisymmetric symbol, $e^{0123} = 1$.

4. Define tensorial differential operators,

$$\chi_{ab} = \gamma_{(a}\gamma_{b)} - \pi_{(a}\pi_{b)} \; = \; g_{ab} - \pi_{(a}\pi_{b)} \,. \tag{2.4}$$

Here one used (1.1).

5. Define a scalar density (differential operator),

$$\chi = \frac{1}{4!}\,e^{abcd}\,e^{efgh}\,\left\{\chi_{ae}\,\chi_{bf}\,\chi_{cg}\,\chi_{dh}\right\}\,. \tag{2.5}$$

6. Define tensorial differential operators, χ^{ab}, satisfying relations,

$$\chi^{ab}\,\chi_{bc} = \delta_c^a \,. \tag{2.6}$$

One may easily find, $\chi^{ab} = g^{ab} + \pi^{(a}\pi^{b)} + O(\ell^4)$, where $\pi^a = g^{ab}\pi_b$.

Scalar densities, reducing to $\sqrt{-g}$ in the limit $\ell \to 0$, are $\sqrt{-\phi}$ and $\sqrt{-\chi}$.[‡] Objects, defined in (2.2) – (2.6), are form-invariant with respect to θ-rotations (1.6) and (1.7).

One may prove a useful formula,

$$e^{abcd}\,e^{efgh}\,\gamma_{[a}\gamma_{e]}\,\gamma_{[b}\gamma_{f]}\,\gamma_{[c}\gamma_{g]} = 10\,g\,\gamma^{[h}\gamma^{d]}\,, \tag{2.7}$$

following from (1.1).[§]

III. ACTION FOR THE FIELDS

1. Consider a system including electron, neutrino, electroweak and gravitational fields, together with scalar fields (quadruplets) U and V. Introduce two spinorial bases. First, L-basis, is represented by octets ($N = 8$), $L_{el} = [\nu_L\,,e_L]^T$, $\overline{L}_{el} = [\overline{\nu}_L\,,\overline{e}_L]$. Here e_L and ν_L are 4-spinors for the left-handed electron and neutrino, respectively; subscript el reminds that respective spinors belong to the electron's flavor. The second, R-basis, is represented by quadruplets, ($N = 4$), e_R and $\overline{e}_R$, corresponding to a right-handed electron.

[‡] One should understand $\sqrt{-\chi}$ as respective expansion in powers of ℓ^2.

[§] Here $\gamma^a = g^{ab}\,\gamma_b$, where $g^{ab}g_{cb} = \delta_c^a$.

2. One may specify the action of π_a on introduced fields.

$$\pi_a L_{el} = -i\ell \left(\partial_a + \frac{i}{2} g' \sigma_k \otimes \hat{1}_4 W_a^k + \frac{i}{2} g'' Y_L \otimes \hat{1}_4 B_a - \hat{1}_2 \otimes \overline{\Gamma}_a \right) L_{el} , \qquad (3.1)$$

$$\pi_a e_R = -i\ell \left(\partial_a + \frac{i}{2} g'' Y_R B_a - \overline{\Gamma}_a \right) e_R , \qquad (3.2)$$

$$\pi_a U = -i\ell \left(\partial_a + \frac{i}{2} g' \hat{1}_2 \otimes \sigma_k W_a^k + \frac{i}{2} g'' Y_W B_a \right) U , \qquad (3.3)$$

$$\pi_a V = -i\ell \, \partial_a V , \qquad (3.4)$$

where B_a and W_a^k ($k = 1, 2, 3$) are potentials of electroweak fields; $Y_L = -\hat{1}_2$, $Y_R = -2 \times \hat{1}_4$, and $Y_W = \hat{1}_4$ are hypercharge operators; σ_k ($k = 1, 2, 3$) are standard (2×2) Pauli matrices; and g' and g'' are interaction constants. Connections $\overline{\Gamma}_a$ (matrices 4×4) pertain to gravitational fields only. $A \otimes B$ denotes the Kronecker product of matrices A and B. From structure of (3.3) follows that under respective matrix transformations components $U' = [u_1, u_2]$ of the scalar $U = [u_1, u_2, u_3, u_4]^T$ do not mix with components $U'' = [u_3, u_4]$ and vice versa, which means that one may consider two independent doublets, $[U', U'']$, combined in U. Dirac matrices, γ_a, have the following structure. In L-basis, $\gamma_a = \text{diag}(\overline{\gamma}_a, \overline{\gamma}_a) = \hat{1}_2 \otimes \overline{\gamma}_a$, where $\overline{\gamma}_a$ are Dirac matrices 4×4. In R-basis, as well as in scalars' U and V bases, $\gamma_a = \overline{\gamma}_a$.

3. To be more specific about the gravitational sector, one may introduce four standard Dirac matrices 4×4, Δ_A, $A = 0, 1, 2, 3$:

$$\Delta_0 = \begin{pmatrix} 0 & \hat{1}_2 \\ \hat{1}_2 & 0 \end{pmatrix} , \quad \Delta_k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix} , \quad k = 1, 2, 3.$$

Then, the following decompositions take place:

$$\overline{\gamma}_a = e_a^A \Delta_A , \qquad (3.5)$$

$$\overline{\Gamma}_a = f_a^{AB} \Delta_{[A} \Delta_{B]} , \qquad (3.6)$$

where e_a^A is a tetrad, and $f_a^{AB} = f_a^{[AB]}$ are six vector fields, which are associated with the Ricci rotation coefficients [19] in standard theory.**

4. One may use the scalar fields $\overline{U}$, U in order to implement symmetry breaking mechanism for generation of masses for vector bosons and fermions. The respective potential energy,

$$V_p(\overline{U}, U, \overline{V}, V) = \sigma_U \overline{U} U + \kappa_U (\overline{U} U)^2 + \sigma_V \overline{V} V , \qquad (3.7)$$

where σ_U, σ_V, and κ_U are constants.

Expanding $\sqrt{-\chi}$ in powers of ℓ, one obtains, $\sqrt{-\chi} = \sqrt{-g} \left(1 - \frac{1}{2} \pi_a \pi^a \right) + O(\ell^4)$. Here $\pi^a = g^{ab} \pi_b$. In the minimum of potential energy (3.7) scalar fields may be taken with the

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** Namely, $f_a^{AB} = \frac{1}{4} e_b^B \nabla_a e^{Ab}$, where ∇_a is a covariant derivative, associated with metric $g_{ab} = e_a^A e_{Ab}$.

following constant values: $U' = [0, p]^T$, $U'' = [0, q]^T$, and $\overline{U}' = [0, p]$, $\overline{U}'' = [0, q]$; one obtains, then,

$$(\overline{U}U)^{-1} \cdot \overline{U}\sqrt{-\chi}U = \sqrt{-g} - \sqrt{-g}\frac{\ell^2}{8}g'^2\left(W_a^1 W^{1a} + W_a^2 W^{2a}\right)$$
$$-\sqrt{-g}\frac{\ell^2}{8}\left(g'W_a^3 - g''B_a\right)\left(g'W^{3a} - g''B^a\right) + O(\ell^2). \tag{3.8}$$

Analogously, one may write an expansion

$$\overline{V}\sqrt{-\chi}V = \sqrt{-g}\left(\overline{V}V + \frac{\ell^2}{2}\overline{V}\partial_a\partial^a V\right) + O(\ell^4). \tag{3.9}$$

5. Similarly, expanding (2.3) into row in powers of ℓ, and using (2.7), one obtains for two spinorial bases the following decompositions. For the L-basis,

$$\sqrt{-\phi_L} = \sqrt{-g} + \frac{1}{48}\sqrt{-g}\,Tr\{\overline{\gamma}^a\overline{\gamma}^b\overline{\rho}_{ab}\}$$
$$+\frac{\ell^4}{320}\sqrt{-g}\left(g'^2 W_k^{ab}W_{ab}^k + g''^2 B^{ab}B_{ab}\right) + O(\ell^4). \tag{3.10}$$

For the R-basis,

$$\sqrt{-\phi_R} = \sqrt{-g} + \frac{1}{48}\sqrt{-g}\,Tr\{\overline{\gamma}^a\overline{\gamma}^b\overline{\rho}_{ab}\} + \frac{\ell^4}{80}\sqrt{-g}\,g''^2 B^{ab}B_{ab} + O(\ell^4), \tag{3.11}$$

where $\overline{\rho}_{ab}$ is constructed via $\overline{\Gamma}_a$ as in (2.1). Only terms pertaining to known components of the standard model are extracted. One uses notations $W_{ab}^k = \partial_a W_b^k - \partial_b W_a^k - g'\epsilon_{lm}^k W_a^l W_b^m$ and $B_{ab} = \partial_a B_b - \partial_b B_a$ for electroweak gauge fields.[††] As shown below, the second term in the right hand side of (3.10) and (3.11) corresponds to EH term.

As it follows from (2.2), (2.3), and (3.4),

$$\sqrt{-\phi_V} = \sqrt{-g}, \tag{3.12}$$

where ϕ_V denotes ϕ (c.f. (2.3)) given in V-representation.

6. Comparing (3.8) - (3.12), one defines action for the fields,

$$S_f = \int d\Omega$$
$$\times \left[\lambda_L\sqrt{-\phi_L} + \lambda_R\sqrt{-\phi_R} - \Upsilon\sqrt{-\phi_V} + \lambda_U\overline{U}\sqrt{-\chi}U + \lambda_V\overline{V}\sqrt{-\chi}V\right], \tag{3.13}$$

where one introduces constants, $\lambda_L, \lambda_R, \lambda_U, \lambda_V$, and Υ, containing the potential (3.7), necessary for symmetry-breaking mechanism:

$$\Upsilon = \lambda_L + \lambda_R + V_p. \tag{3.14}$$

[††] Indices k, l, m ... are raised and lowered with $\delta_{km} = \delta^{km} = diag(1, 1, 1)$.

The action (3.13) together with the action for spinors ((4.3) below) should be varied with respect to fields e_a^A, f_a^{AB}, W_a^k, B_a, $\overline{U}$, U, $\overline{V}$, V, $\overline{L}_{el}$, L_{el}, $\overline{e}_R$, and e_R.

Comparing the expansion of (3.13) in ℓ with the standard action, one obtains the following expressions for constants:

$$\ell^2 = \frac{10}{3\alpha}(1 + 2\sin^2\theta_W)\,\ell_p^2\,; \tag{3.15}$$

$$\lambda_L = -\frac{6}{\pi}\frac{\sin^2\theta_W}{1 + 2\sin^2\theta_W}\frac{c^3}{k\ell^2}\,; \tag{3.16}$$

$$\lambda_R = -\frac{3}{2\pi}\frac{1 - 2\sin^2\theta_W}{1 + 2\sin^2\theta_W}\frac{c^3}{k\ell^2}\,; \tag{3.17}$$

$$\lambda_U = -\frac{1}{8\pi c\ell^2}\,; \tag{3.18}$$

$$\lambda_V = -\frac{1}{8\pi c\ell^2}\,; \tag{3.19}$$

$$p^2 + q^2 = 4\sin^2\theta_W\,\frac{m_W^2 c^4}{e^2}\,, \tag{3.20}$$

where α is the fine structure constant for an electron, θ_W is Weinberg's mixing angle [20], ℓ_p is Planck's length, and m_W is vector bosons' rest mass.

Note, that scalar field's V sole purpose is to make invariant (3.12) possible; in turn, one needs $\sqrt{-\phi_V}$ in order to adjust the 'cosmological term', so that it would have a reasonable value. The field V doesn't interact with other fields. On the other hand, it contributes stress-energy to the Einstein equations; thus one may think of it as of the 'dark matter'.

IV. ACTION FOR SPINORS

The relevant tensors, invariant with respect to (1.6) and (1.7), are:

$$\phi^{dh} = \frac{4}{5!\phi}e^{abcd}e^{efgh}\,\phi_{ae}\,\phi_{bf}\,\phi_{cg}\,; \tag{4.1}$$

$$\Sigma_{ab} = \gamma_{[a}\pi_{b]} - \pi_{[a}\gamma_{b]}\,. \tag{4.2}$$

Then, the action for fermions may be constructed as follows:

$$S_\Psi = C \int d\Omega$$
$$\times \left[\sqrt{-\phi_L}\,\overline{L}_{el}\,\phi^{ab}\,\Sigma_{ab}L_{el} + \sqrt{-\phi_R}\,\overline{e}_R\,\phi^{ab}\,\Sigma_{ab}e_R + r\sqrt{-\phi_V}\left(\overline{L}_{el}e_R U' + \overline{U}'\overline{e}_R L_{el}\right)\right]\,. \tag{4.3}$$

Expanding (4.3) in powers of ℓ, one obtains,

$$S_\Psi = \frac{2\ell C}{\hbar}\int d\Omega\sqrt{-g}\left[\overline{L}_{el}\,\gamma^a p_a L_{el} + \overline{e}_R\,\overline{\gamma}^a p_a e_R + \frac{\hbar r p}{2\ell}\left(\overline{e}_L e_R + \overline{e}_R e_L\right)\right] + \cdots \tag{4.4}$$

The momentum operator, $p_a = -i\hbar(\partial_a - \Gamma_a)$, is given in respective spinorial bases.

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V. GRAVITATIONAL SECTOR

It will be shown, that the gravitational part of the action (3.13) in vacuum (in the limit $\ell \to 0$) is equivalent to the EH action in general relativity. First, extract the action of the gravitational field from the expansion of (3.13) in powers of ℓ:

$$S_g = \kappa^{-1} \int d\Omega \sqrt{-g} \, Tr \left\{ \overline{\gamma}^a \, \overline{\gamma}^b \, \overline{\rho}_{ab} \right\} , \tag{5.1}$$

Varying (5.1) with respect to $\overline{\gamma}_a$ and $\overline{\Gamma}_a$, one obtains the equations,

$$\left[\overline{\gamma}^b , \overline{\rho}_{ab} \right] - \frac{1}{4} \overline{\gamma}_a \, Tr \{ \overline{\gamma}^c \, \overline{\gamma}^d \, \overline{\rho}_{cd} \} = 0 , \tag{5.2}$$

$$\frac{1}{\sqrt{-g}} \, \partial_b \left(\sqrt{-g} \, \overline{\gamma}^{[a} \, \overline{\gamma}^{b]} \right) - \left[\overline{\Gamma}_b , \overline{\gamma}^{[a} \, \overline{\gamma}^{b]} \right] = 0 . \tag{5.3}$$

Equations (5.3) are equivalent to the following:

$$\left(\overline{\gamma}^{[a} \, \overline{\gamma}^{b]} \right)_{;b} = 0 , \tag{5.4}$$

where the semicolon denotes covariant derivative. For the covariant derivative of $\overline{\gamma}_a$ one obtains,

$$\overline{\gamma}_{a\,;\,b} = \frac{i}{\ell} \left[\pi_b, \overline{\gamma}_a \right] = \partial_b \overline{\gamma}_a - \Gamma^c_{ab} \overline{\gamma}_c - \left[\overline{\Gamma}_b , \overline{\gamma}_a \right] . \tag{5.5}$$

From (5.4) follows (as one possible solution),

$$\overline{\gamma}_{a\,;\,b} = 0 . \tag{5.6}$$

Applying the antisymmetrized product of covariant derivatives to $\overline{\gamma}_b$ and using (5.6), one obtains,

$$R^a_{bcd} \overline{\gamma}_a = \frac{1}{\ell^2} [\overline{\gamma}_b , \overline{\rho}_{cd}] , \tag{5.7}$$

where R^a_{bcd} is the Riemann tensor,

$$R^a_{bcd} = 2 \left(\partial_{[c|} \Gamma^a_{b|d]} - \Gamma^e_{b[c|} \Gamma^a_{e|d]} \right) . \tag{5.8}$$

For the scalar curvature and Ricci tensor one obtains, respectively,

$$R = \frac{1}{2\ell^2} Tr \left\{ \overline{\gamma}^a \, \overline{\gamma}^b \, \overline{\rho}_{ab} \right\} , \tag{5.9}$$

$$R_{bd} = \frac{1}{4\ell^2} Tr \left\{ \overline{\gamma}^a [\overline{\gamma}_b , \overline{\rho}_{ad}] \right\} . \tag{5.10}$$

In given approximation (i.e., in the limit $\ell \to 0$), it follows from (5.7),

$$\overline{\rho}_{cd} = -\frac{\ell^2}{4} R_{abcd} \, \overline{\gamma}^a \overline{\gamma}^b . \tag{5.11}$$

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Here, again, $\overline{\gamma}^a = g^{ab}\overline{\gamma}_b$. Multiplying (5.2) by $\overline{\gamma}_c$ and taking the trace, one obtains the vacuum Einstein equations,

$$R_{ac} - \frac{1}{2}g_{ac}R = 0\,, \tag{5.12}$$

where definitions (5.10) and (5.9) are used for the Ricci tensor and scalar curvature in '$\overline{\rho}$–$\overline{\gamma}$' representation. Note that on the classical level, when the matter lagrangian doesn't depend on $\overline{\Gamma}_a$, equations (5.12) are still valid, if one adds the stress-energy tensor for matter to the right hand side of (5.12).

VI. MEANING OF PARAMETER θ

From the structure of the action for the electron (4.4), follow expressions for the correction to the energy, δE, due to the value of the potentials of electroweak fields at spatial infinity. Namely,

$$\delta E_L = \frac{\hbar}{2}\left(g'W_0^3(\infty) + g''B_0(\infty)\right)\int dV\,\overline{e}_L\overline{\gamma}^0 e_L\,; \tag{6.1}$$

$$\delta E_R = \hbar g''B_0(\infty)\int dV\,\overline{e}_R\overline{\gamma}^0 e_R\,. \tag{6.2}$$

One uses notations, $W_0^3(x^a) \to W_0^3(\infty)$, as $|x^a| \to \infty$, etc. From (1.6) and (1.7) follows,

$$\int \overline{L}_{el}\gamma'^a d\Sigma_a L_{el} = \cosh\theta \int \overline{L}_{el}\gamma^a d\Sigma_a L_{el} + \sinh\theta \int \overline{L}_{el}\pi^a d\Sigma_a L_{el}\,, \tag{6.3}$$

$$\int \overline{L}_{el}\pi'^a d\Sigma_a L_{el} = \sinh\theta \int \overline{L}_{el}\gamma^a d\Sigma_a L_{el} + \cosh\theta \int \overline{L}_{el}\pi^a d\Sigma_a L_{el}\,. \tag{6.4}$$

One uses notations, $\gamma'^a = \chi^{ab}\gamma'_b$, $\gamma^a = \chi^{ab}\gamma_b$, $\pi'^a = \chi^{ab}\pi'_b$, and $\pi^a = \chi^{ab}\pi_b$ One integrates over an arbitrary hypersurface with $d\Sigma_a = \sqrt{-\chi}\,e_{abcd}\,dx^b\,dx'^c\,dx''^d$. One may denote energy, $E = \int \overline{L}_{el}\,p^0 dV\,L_{el}$, and electrical charge, $Q = \int \overline{e}_L\overline{\gamma}^0 dV\,e_L$, where $dV \equiv d\Sigma_0$ is the element of spatial volume, and $p^a = \frac{\hbar}{\ell}\pi^a$ is the momentum operator. Selecting spatial volume as a hypersurface of integration, one obtains

$$E' = \cosh\theta\,E + \frac{\hbar}{\ell}\,\sinh\theta\,Q\,; \tag{6.5}$$

$$Q' = \cosh\theta\,Q + \frac{\ell}{\hbar}\,\sinh\theta\,E\,. \tag{6.6}$$

Equations (6.5) and (6.6) represent transformations of energy and electrical charge, respectively. Suppose, that $E = 0$. Then, $\tanh\theta = \frac{\ell}{\hbar}\frac{E'}{Q'}$, which means that energy E' is due as a whole to electrical charge Q', placed in constant electroweak potential, as in (6.1). Thus,

$$\tanh\theta = \frac{\ell}{2}\left(g'W_0^3(\infty) + g''B_0(\infty)\right)\,. \tag{6.7}$$

Analogous considerations for e_R instead of L_{el} lead to the equation

$$\tanh\theta = \ell g''B_0(\infty)\,, \tag{6.8}$$

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from which it follows that one should put $g'W_0^3(\infty) = g''B_0(\infty)$.[‡‡] Thus, one should associate the θ-transformation with that of electroweak potentials at spatial infinity. From (6.7) it follows that $\theta \propto \ell$, and neglecting terms of order ℓ^2 in (6.6), one obtains $Q' \approx Q$.

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[‡‡] One should use $\sin\theta_W\, g' = \cos\theta_W\, g'' = \frac{e}{\hbar c}$.

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